**Strong Disorder**

If we go to higher order in 1/kF­ℓ, say by considering maximally crossed diagrams (see Conductivity calculations), then we should somehow figure out that at least in D < 2, and for L < Lφ, the wave-like character of the eigenstates is completely destroyed. Instead we have localized wavefunctions. Perhaps we could only come to this conclusion by looking at transport functions. We might also suspect something of the sort considering that only in D < 2 do arbitrarily small V wells support bound states. Either way, if we do look at transport functions, like diffusion and conductivity, we will see that states will indeed be localized with an exponential envelope, ψ(r) ~ exp(-r/ξ), where ξ is the localization length.

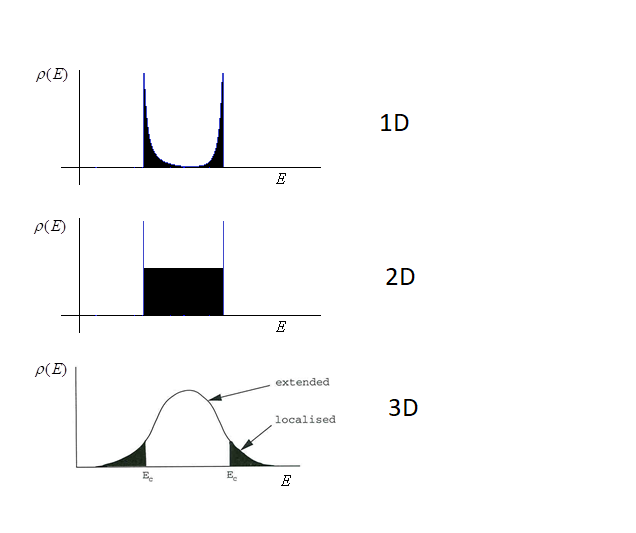


And we get something like:

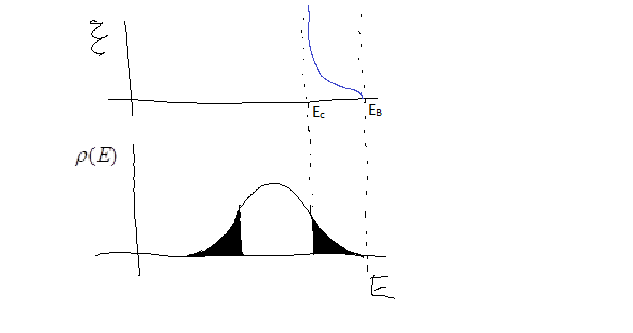


where b is the transverse dimension, and disorder W = 1/kℓ = λ/ℓ. Note the localization length can be much larger than the length of the sample, especially in 2D, and so the wavefunctions’ localization may not be readily apparent. We see that ξ depends on the dimension of the sample, generally getting larger as D increases, and in fact being ∞, if disorder is not large enough, when D > 2 as it turns out. This has to do with the ‘probability of self-intersection’ as we’ll see later. Then it is also larger as τ gets larger, which makes sense. And also larger as k gets smaller, which we might suspect has to do with the fact that larger kinetic energies, or velocities, are harder to localize. Might also relate this to the fact that smaller k relates to a larger ‘internal pressure’ in the Bohm wavefunction picture, and so this also helps explain the greater difficulty in localizing such a particle.

In general we’d expect that all things disorder-wise being equal, ξ would be finite everywhere in the band for D < 2, but be smallest near the band edges (where <v> = 0), and get larger as we approach the center (middle energy) of the band (where <v> is largest). Likewise in D > 2, ξ is finite perhaps near the band edges (see DOS section below as might be rather that it takes a finite disorder to even localize the extreme band edge), and likewise increases as we approach the center of the band, and eventually diverging to ∞ at some critical value kc (or Ec depending on your preference) for a given disorder strength τ. This defines the so-called mobility edge – and will remain ∞ all inside the mobility window. This is illustrated below (the band edges of the 1,2D spectrum isn’t *supposed* to line up with Ec in 3D, it just happened that way):



As we approach the critical energy we would expect something like this: ξ ~ 1/(EC – E)ν. Variously ν = ½ or 1 have been reported. The latter value might be due to magnetic effects. On the other hand, near the band edge, I think he says that according to the tight binding model, ξ goes as ξ ~ (EB – E)2. So it roughly looks like this:



Aside from the <v> **velocity point of view** of explaining how ξ changes within the band, we can examine the situation from an analogous **position point of view**, we’ll observe that these curves are the density of states *per unit volume*. It is peaked about the middle of the energy band spectrum. As disorder is increased, electron transport will occur via tunneling. States in the band tales are likely to be bound by the fluctuating potential because to tunnel through to another state with significant probability would require that state be of similar energy (there is no mechanism for gaining or losing energy at T = 0, and no e-e interactions). But these states are likely widely spaced, as evinced by their low density. Therefore the probability of tunneling is relatively small, and they remain localized. States near the center are likelier to be extended by the same arguments. The probability of tunneling from from one site to another is proportional to the probability that the two sites have the same energy.

**DOS**

Of course, to presume that the density of states keeps the same form regardless of disorder isn’t accurate either. So this picture isn’t quite right. At W = 0 we’ll have the unperturbed free density of states. In fact, as disorder begins to increase at first, the W fluctuations broaden the density of states – and continues to as W increases. This is because the impurity potential will mix states of different energy (perturbation theory). And it was shown that the Van Hove singularities are smoothed out. And shown that in between the ‘gaps’ g(E) is still finite, but super-exponentially small. Using the tight-binding models especially, one can show that in the presence of disorder, the density of states doesn’t have any Van Hove singularities, i.e. abrupt drops to 0. They do go to zero and there are band gaps still but they go to zero in a continuous way, given by:



where V+/- are the end point of the band. So they go to zero as exp(-1/0).



So as disorder increases, the density of states broadens. We see that at W = 0, the entire band is conducting (since the band ranges in energy between -6 and 6 in his model). And as long as W is weak the states in the tails may still conduct. But as disorder increases, the states in the tails of the distribution (still widening) will be localizing, and the mobility edge will begin to begin inwards towards thte energy band center. Eventually the edge will cross the Fermi surface and σ will be shut off. I will suppose that for his model purposes below, the Fermi energy is at E = 0 (in the middle of the band). Of course if we’re talking about the middle of the band, then this is where all states get shut off. So at W = 16.5 there are no more conducting states in the band.



Note that this means that all eigenstates of the Hamiltonian are localized – for any random distribution of impurities with that critical disorder strength.